The 2C and 3C Controllers: Multirate Single-Loop Digital Logic

Daokui Jiang
Thomas J. Ward
Chemical Engineering Department
Clarkson University
Potsdam, NY 13676

A careful study of skilled operators running a process plant under manual control led to the notion of multirate digital control structures that have the superficial appearance of digital PI and PID controllers. However, the control action is quite different. The resulting multirate controllers are introduced here as the 2C and 3C controllers.

This paper presents a discussion of the 2C and 3C controllers, a practical tuning guide based on input-output tests, and comparative simulation results.

Background

Early process controllers were attempts to mimic the efforts of skilled plant operators. While the operator actions were intermittent, the early controllers were continuous in nature. The frequent operator changes based on the observed instantaneous error led naturally to the continuous proportional and rate modes of a controller. In addition, there was a relatively infrequent operator action to manually reset the controller for persistent error. The integral mode was invented as a continuous action to correspond to this infrequent reset action.

An early process control alternative was the damped impulse control mode (Grebe et al., 1933), which provided a corrective change in addition to that of the proportional and rate changes, but would then "wait for the effect of this change before continuing to operate." It was noted that this was the "equivalent of an operator . . . waiting to see what further correction will be required." This novel approximation to the operator reset response apparently was abandoned when commercial vendors offered the continuous reset (integral) mode as a more economical solution.

The analog single-loop PID structure, with interactive modes, served as the basis of most process control systems until digital computer control systems were introduced. However, the flexibility and power of digital systems has not been fully exploited and much digital process control is still based on the single-loop PID theme, with difference approximations used in some separable mode algorithms.

Multirate digital control has had a long history in nonprocess areas. This has been reviewed extensively (Walton, 1981; Glasson, 1983). Both frequency domain (Ragazzini and Franklin, 1958; Whitbeck, 1978; Rattan, 1981) and optimal time domain techniques (Broussard and Glasson, 1980) have been developed. Recent efforts are typified by Kando and Iwazumi (1986) and Araki and Hagiwara (1986). The early motivation for this work, largely in aerospace systems, was the problem posed by different sensor rates. However, a fertile area of application is the simultaneous existence of fast and slow elements with large differences in time constants or bandwidths (Litkouhi and Khalil, 1985). While this combination appears to be fairly common in process systems, the actual process structures and parameters are usually not well identified. For this reason, the application of these reported multirate methodologies to process systems appears to be quite limited.

The 2C and 3C Controller

Jiang (1977) observed that skilled manual operation of the principal control loop in a Chinese processing unit was consistently more effective than well-tuned automatic control. He then made a careful analysis of the actions of the most effective operators. He noted that the operators made incremental changes proportional to the "changed in error" after each error measurement, which was done frequently and regularly. If the error was persistent after some time, a large incremental change was made to "calibrate" the controller. This relatively infrequent calibration correction was roughly proportional to the magnitude of the error. After "calibrating" the controller, the operators would not recalibrate until they had a change to observe the effect of the last calibration. This meant that there was a waiting period at least as large as the process delay time before recalibration. Jiang used these observations to develop a

The present address of D. Jiang is Southwest Research Institute for Chemical Industries, Noxi, Sichuan, China.

digital controller that would mimic the "patient" operator rather than the conventional analog controller.

These observations can be incorporated into a process control velocity algorithm knonw as the 2C controller, with an "action" and a "calibration" mode:

$$\Delta m_n = C_A(e_i - e_{i-1}) + C_C(e_j) \quad (1)$$

where C_A is called the action mode gain, C_C is the calibration mode gain, Δm_n is the controller output increment at the nth sampling instant, and e is the process error. The action mode sampling period, T_A, is given by

$$T_A = (t_i - t_{i-1}) \quad (2)$$

and the calibration-mode sampling period, T_C, is

$$T_C = (t_j - t_{j-1}) \quad (3)$$

Both of these mode sampling periods are assumed to be integer multiples of the process output sampling period.

The 2C controller of Eq. 1 is similar in form to the PI velocity algorithm with a rectangular integrator, given in Eq. 4:

$$\Delta m_n = K_c(e_n - e_{n-1}) + (K_c T_s/T_i)e_n \quad (4)$$

This might suggest that the 2C controller is simply a multirate PI controller. However, the 2C controller has FOUR adjustable parameters (C_A, C_C, T_A, T_C), different tuning requirements, and quite different control action. It has been given the 2C name to emphasize this difference and to identify that it is a two-coefficient controller.

It might seem logical to extend this approach to a three-mode controller analogous to the three-mode PID controller. Indeed, Jiang's analysis of the plant operators suggested that an additional response action was present in some cases, where the incremental change in manipulative input was a function of the rate of change of the sampled error increment (the second derivative of the sampled error). The corresponding 3C controller can be written as

$$\Delta m_n = C_A(e_i - e_{i-1}) + C_C e_j + C_R(e_k - 2e_{k-1} + e_{k-2}) \quad (5)$$

where C_R is the rate mode gain. The rate mode period, T_R, is given by

$$T_R = t_k - t_{k-1} \quad (6)$$

This is similar in form to the PID velocity algorithm with a rectangular integrator and a two-point differentiator, given in Eq. 7:

$$\Delta m_n = K_c(e_n - e_{n-1}) + (K_c T_s/T_i)e_n + (K_c T_d/T_s)(e_n - 2e_{n-1} + e_{n-2}) \quad (7)$$

However, the 3C controller has SIX adjustable parameters and a different control action. As an illustration of this difference, consider the open-loop response of the PID and 3C controllers to the hypothetical error input shown in Figure 1. In this figure, all

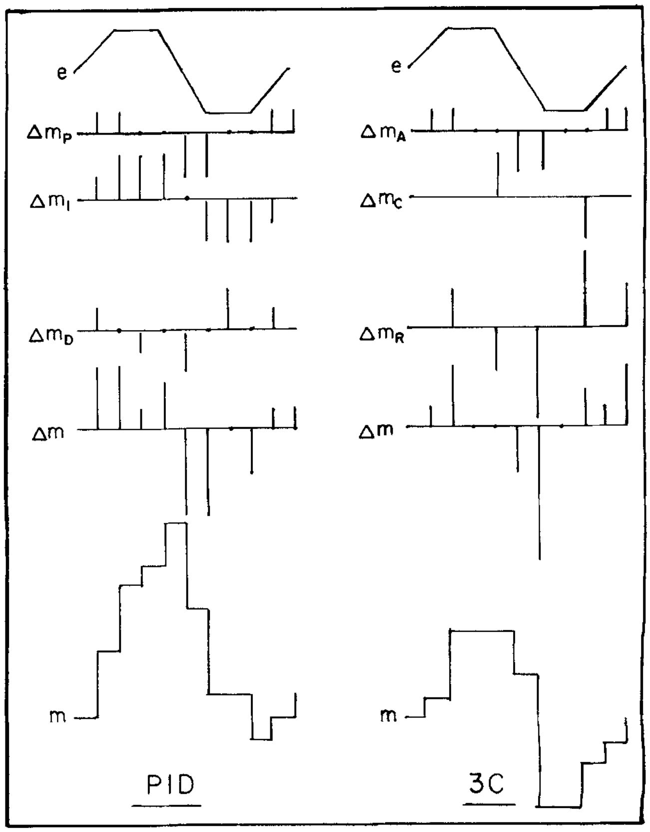

Figure 1. Output of the PID and 3C controllers to a hypothetical error function.
Calibration-mode sampling period = 4, rate-mode sampling period = 2, all other controller parameters unity. The incremental outputs associated with each mode are also shown.

mode coefficients, the error sampling rate, and the action-mode period are set equal to unity, but the mode periods for the calibration and rate modes of the 3C controller are selected as 4 and 2, respectively. The responses of the individual modes and overall controller output are shown for each controller.

A simulated comparison was made to illustrate the improved response of the 2C and 3C controllers over the conventional PI and PID controllers. The first-order/delay process model, with gain, $K = 1$, time constant, $T = 100$ s, and a range of delay times, was selected. A unit step input disturbance was used in all cases and the output sampling period was one second. The PI and PID controllers (Eqs. 4 and 7) were tuned to give arbitrary "optimal" responses that correspond to quarter-decay tuning. For comparison, the 2C and 3C controllers (Eqs. 1 and 5) were turned to give the *same initial peak height* with as much attenuation of the second peak as possible.

Typical results of this preliminary comparison are shown in Figures 2 and 3. In all cases, the 2C and 3C controllers gave significantly greater attenuation and shorter settling times, although the period of oscillation is often slightly greater. It should be mentioned that the integral- and calibration-mode coefficients ($K_c K_s/T_i$ and C_C) are quite different, particularly as the τ/T ratio increases, and that the calibration-mode period, T_C, is always greater than the delay time. This corresponds to the "patient operator" notion mentioned earlier (Grebe et al., 1933; Jiang, 1979).

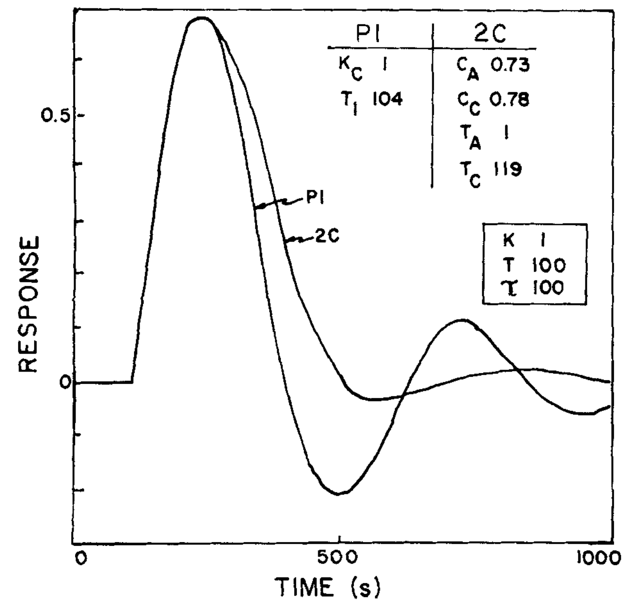

Figure 2. Response to disturbance with PI vs. 2C controllers.
Sample time = 1 s.

Development of Tuning Guides

The process model selected for the tuning guide development was the first-order/delay type, with gain, K, time constant; T, and delay, τ. While this will represent only overdamped process characteristics, it was selected because it is the basis of numerous analog tuning guides used in the process industries. The upset for the simulation studies was selected as a unit step process disturbance input. Thirty-two sets of process conditions were selected for evaluation in the simulations. These provided a K range from 0.5 to 2.5, a T range from 60 s to over one hour, and τ/T ratios from 0.01 to 1.0.

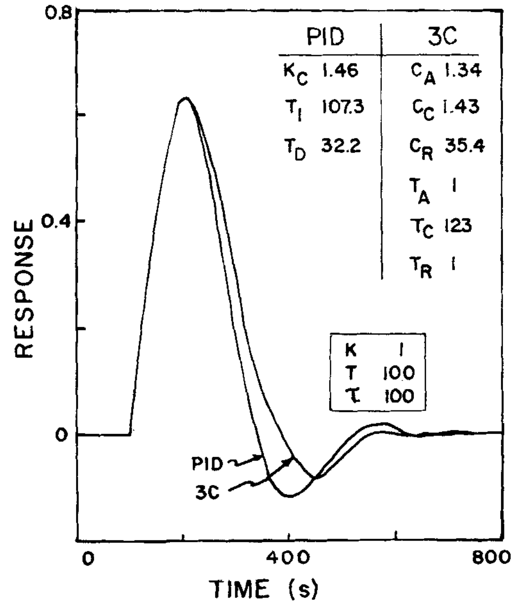

Figure 3. Response to disturbance with PID vs. 3C controllers.
Sample time = 1 s.

The response criteria that were selected for the parameter search were based on the typical response curve shown in Figure 4. The three response criteria were as follows:

- $|P_1| \geq 2|P_2|$
  $|P_2| \geq 2|P_3|$
  $|P_3| \geq 2|P_4|$
  $|P_4| \geq 2|P_5|$

- Make P_1 as small as possible while satisfying the first criterion.
- Keep this minimum P_1 and search for the maximum attenuation ratio, P_1/P_3.

These criteria were selected to give a stable, reasonably damped response and a unique set of controller parameters. No consideration was given to settling time.

These rules served as the basis for the development of a search routine that could evaluate several hundred controllers for each process case. During the exploratory phase of the parameter search, a decision was made to fix three parameters so as to reduce the scope of the search. The process sampling time was selected as 0.5 s. This represents a reasonable compromise between accuracy, speed of response, and truncation error. The action-mode sampling period, T_A, and rate-mode sampling period, T_R, were both fixed at one second. This still preserves the concept of the "patient operator."

The results for each process case were analyzed and some further fine-tuning studies were made to improve response characteristics. The final controller settings for each case were then correlated as graphical functions of the process parameters and fitted to simple piecewise functions.

The 2C Tuning Guide

The recommended 2C parameter values based on the first-order/delay process model have been correlated as the following

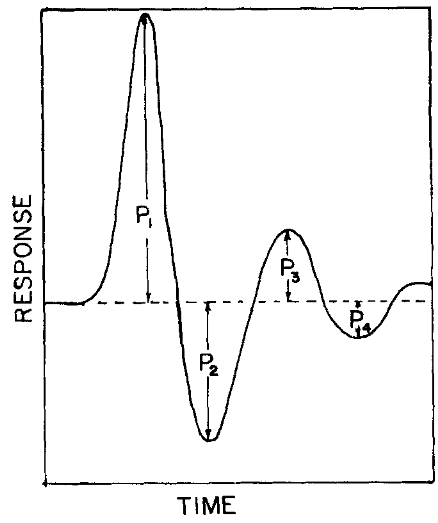

Figure 4. Typical controlled response to a unit step disturbance.

tuning guide:

Action-mode gain, C_A

$$C_A = \begin{cases} (1.68/K)\,(\tau/T)^{-0.715} & 0.005 \le \tau/T \le 0.05 \\ (0.942/K)(\tau/T)^{-.909} & 0.05 \le \tau/T \le 1.0 \end{cases}$$

Action-mode sampling period, T_A

$T_A = 1$ s)

Note: When the time constant, T, is less than 100 s, then T_A = 0.5 s is probably a better choise if the process sampling period is $\le$0.5 s. If T is very large, then a larger value of T_A might be desirable to improve digital resolution.

Calibration-mode gain, C_C

$$C_C = (0.810/K)(\tau/T)^{-0.728} \quad 0.005 \le \tau/T \le 1.0$$

Calibration-mode sampling period, T_C

$$T_C = \begin{cases} 1.5 \text{ (and round up)} & 0.005 \le \tau/T \le 0.05 \text{ (or } \tau \le 5\text{s)} \\ (1.15)(\tau/T)^{-0.123} & 0.05 \le \tau/T \le 1.0 \text{ (or } \tau > 5\text{s)} \end{cases}$$

The 3C Tuning Guide

The recommended 3C controller parameters based on the first-order/delay process model lead to the following 3C tuning guide:

Action-mode gain, C_A

$$C_A = \begin{cases} (13.0K)(\tau/T)^{-0.283} & 0.005 \le \tau/T < 0.01 \\ (3.45/K)(\tau/T)^{-0.570} & 0.01 \le \tau/T < 0.2 \\ (1.38/K)(\tau/T)^{-1.14} & 0.2 \le \tau/T \le 1.0 \end{cases}$$

Action-mode sampling period, T_A

($T_A = 1$ s)

Note: When the time constant, T, is less than 100 s then T_A = 0.5 s is probably a better choice if the process sampling period is $\le$0.5 seconds. If T is very large, then a larger value of T_A might be desirable to improve digital resolution.

Calibration-mode gain, C_C

$$C_C = \begin{cases} (7.65K)(\tau/T)^{-0.204} & 0.005 \le \tau/T < 0.03 \\ (3.95/K)(\tau/T)^{-0.392} & 0.03 \le \tau/T < 0.2 \\ (1.32/K)(\tau/T)^{-1.071} & 0.2 \le \tau/T \le 1.0 \end{cases}$$

Calibration-mode sampling period, T_C

$$T_C = \begin{cases} 1.5 \text{ (and round up)} & 0.005 \le \tau/T < 0.05 \text{ (or } \tau < 5s) \\ (1.16)(\tau/T)^{-0.077} & 0.05 \le \tau/T \le 1.0 \text{ (or } \tau > 5s) \end{cases}$$

Rate-mode gain, C_R

$$C_R = \begin{cases} (8.10/K)(T/T_R)(\tau/T)^{0.849} & 0.005 \le \tau/T < 0.03 \\ (1.03/K)(T/T_R)(\tau/T)^{0.265} & 0.03 \le \tau/T < 0.3 \\ (0.302/K)(T/T_R)(\tau/T)^{-0.650} & 0.3 \le \tau/T \le 1.0 \end{cases}$$

Rate-mode sampling period, T_R

($T_R = 1$ s)

Note: When $T < 100$ s, then $T_R = 0.5$ is suggested if the error sampling period is $\le$0.5 s. If noise is significant, then a larger T_R might be more effective.

Typical Results

Typical results obtained during the development of the tuning guides are shown in Figure 5. In general, the results demonstrated that the tuning guides will provide stable, damped responses to disturbance inputs for processes that can be characterized by a first-order/delay model.

As with any tuning guide, further fine tuning is often desirable. For example, the response with a 3C controller in Figure 5, shown as a dashed line, might be too oscillatory in some applications. Damping can be increased by decreasing C_A or increasing C_R. An increase in C_R from 159 to 180 gave the 3C response, shown as a solid line. Since the action and rate modes correspond closely to the conventional proportional and derivative modes, experienced operators should be able to make effective small corrections of this type in the controller parameters so as to obtain better response.

A brief simulation study was carried out to study the effect of model errors on these controller algorithms. Each of the three process parameters (K, T, and τ) was changed in turn by $\pm$10% from the values used in specifying the controller parameters. This 10% error in each of the parameters still led to stable responses for all the process cases studied, although there was more oscillation in the responses. As might be expected, errors in the delay time, τ, produced the most significant changes. Typical results are shown for one process case with a 3C controller in Figure 6.

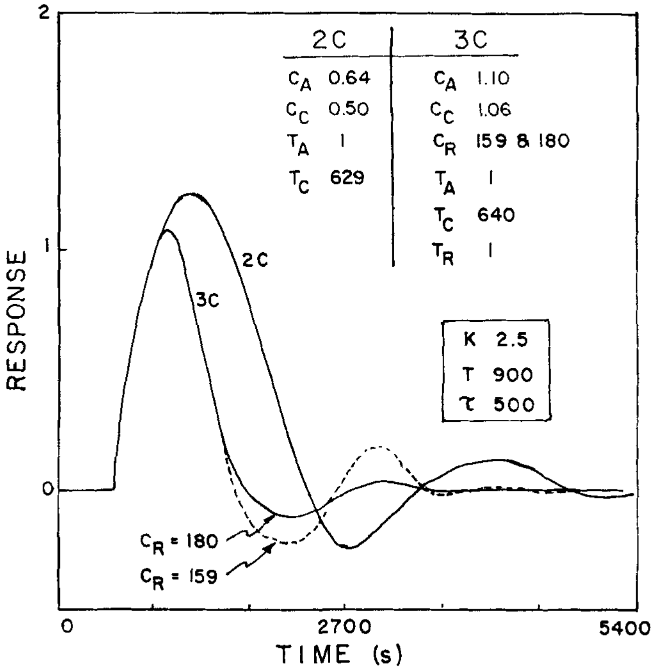

Figure 5. Typical tuned responses of first-order/delay process to unit step disturbance input.

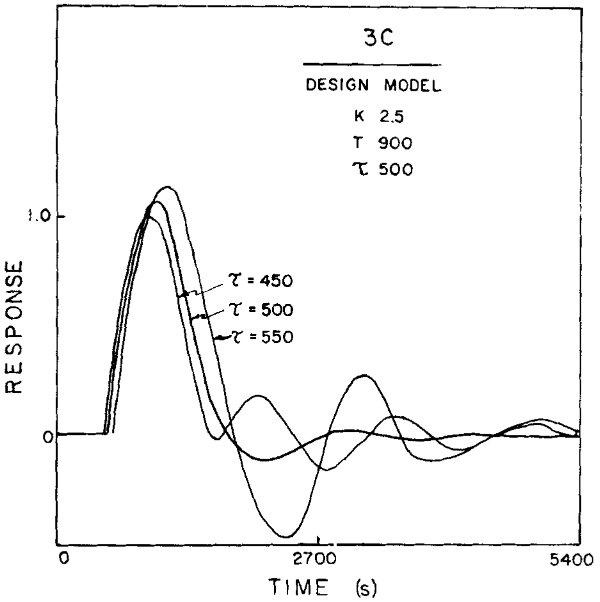

Figure 6. Effect of time delay error on a 3C-controlled process.

Discussion

The 2C and 3C controllers were designed to mimic the patient process operator who waits to see the effect of his periodic reset or calibration action before he attempts to reset the controller. This shows up in the tuning guides as a sampling period, T_C, for the calibration mode that always is larger than the delay time. Since the calibration mode makes infrequent changes in the controller output, the gain, C_C, of this mode is relatively large compared to the gain of the integral mode in conventional PI or PID algorithms. In other words, the calibration mode consists of large, infrequent contributions while the digital integral mode is characterized by small, but frequent contributions.

As noted above, the sampling periods, T_A and T_R, for the two other controller modes were selected arbitrarily to simplify the study. Different values of these parameters might be more effective in some cases, but this would require a more extensive simulation study.

The action-mode gain setting, C_A, corresponds roughly to the proportional gain recommendations that would be obtained from several PI and PID tuning guides. However, the rate-mode gain, C_R, does not correspond to the derivative-mode recommendations. This is probably due to the influence of the calibration mode.

If the process τ/T ratio is greater than one, the control of the process with the 2C or 3C controller degrades and a model-based delay compensator such as the Smith predictor should be used.

Various modifications of the 2C and 3C algorithms are possible. Other integration schemes, high-order derivative approximations, derivative filters, and output-actuated action and rate modes would be possibilities for further study.

One likely difficulty that has not been explored in detail is process noise. However, some preliminary evaluation of these algorithms in experimental process cases with noise (laboratory-scale level control loop, pilot-scale distillation column with four loops, and an analog-simulated second-order process with delay) indicate that the performance is better than with conventional controllers. A plant-scale evaluation of these algorithms in China led to a 6% increase in productivity when compared with conventional control algorithms.

A separate tuning guide for setpoint inputs has not been developed. A brief simulation study of several process cases tends to indicate that the above tuning guides are satisfactory for setpoint changes.

The 2C and 3C controllers probably could be incorporated into autotuner options without much difficulty. They would not be suitable for most adaptive approaches, which tend to generate a polynomial ratio type of model.

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